CHAPTER 11

Piles of Tiles

Investigating Patterns with Color Tiles

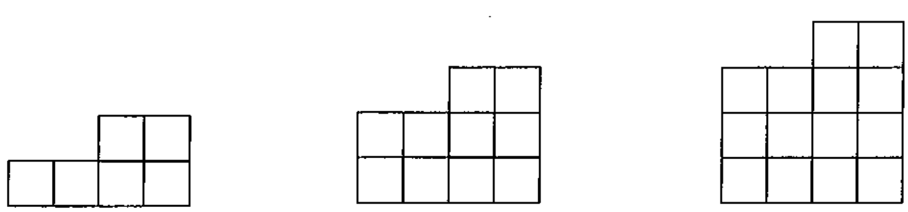
OVERVIEW

In this lesson, students use square tiles to make and check predictions about geometric patterns. After seeing the first three piles of tiles in a pattern, the students predict what the fourth and fifth piles look like. They record on T-charts, write about the patterns they discover, and represent the patterns with equations and graphs. The arrangements of tiles chosen for this lesson typically evoke several geometric interpretations from students, but the numerical patterns are the same for each interpretation.

BACKGROUND

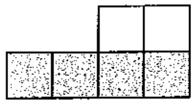
Using square tiles gives students a concrete way to interact with patterns that grow. Also, investigating the growing shapes gives students experience connecting geometry, number, and algebra.

Students first see the following three piles of tiles:



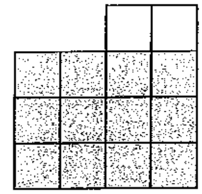
Pile 1 **Pile 2** **Pile 3**

When they think about extending the pattern beyond the first three piles, they typically offer several interpretations, as occurred in this class. Several ways are valid. Some students focus on the rows of four and then add the two extra tiles on the top. In this interpretation, they notice that the number of rows is the same as the pile number.



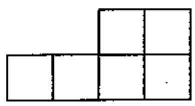
Pile 1
1 row of 4

Pile 2
2 rows of 4

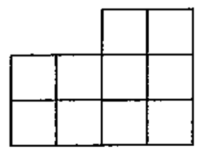


Pile 3
3 rows of 4

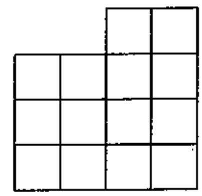
Other students focus on the columns and see two shorter columns on the left and two longer columns on the right. They notice that the number of tiles in the shorter columns is the same as the pile number, and the number of tiles in the longer columns is one more than the pile number.



Pile 1
2 columns of 1
2 columns of 2

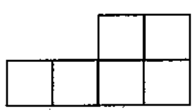


Pile 2
2 columns of 2
2 columns of 3

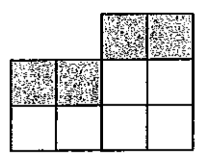


Pile 3
2 columns of 3
2 columns of 4

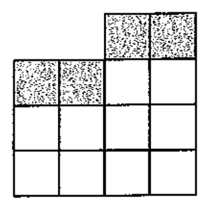
Some students see that the next pile in the pattern can be constructed by adding a tile to the top of each column in the previous pile. Some think of this as adding two rows of two.



Pile 1

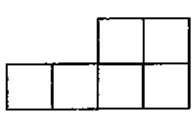


Pile 2
4 tiles added
to top of Pile 1

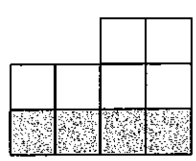


Pile 3
4 tiles added
to top of Pile 2

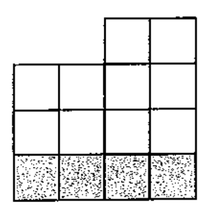
Other students also look to the previous pile but think of adding four tiles to the bottom of it to get the next one.



Pile 1



Pile 2
4 tiles added
to the bottom
of Pile 1



Pile 3
4 tiles added
to the bottom
of Pile 2

In the lesson, students use the patterns they discover to predict how the tenth and hundredth piles will look. They also write equations to represent the patterns and use the ordered pairs on the T-chart to plot points to make a coordinate graph. While the students can explore the tile patterns with no prerequisite preparation, they should have had experience writing equations and plotting points before being asked to do so for this lesson. The class in the vignette had previous experience with both.

Extending the pattern backward in the T-chart presents the opportunity to talk with the students about the distinction between looking at the patterns mathematically by focusing only on the numerical data and looking for patterns in the numbers, or looking at the pattern in relation to the concrete situation of building with tiles. For example, it's possible to extend the pattern back and see that the ordered pair (–1, –2) fits the pattern of the other pairs of numbers on the T-chart. However, it's not possible to build a pile with a negative number of tiles. While we can talk about the numbers and plot the points on a graph, there's a "loss of reality" when relating those numbers to the problem-solving situation.

During the course of the lesson, students use variables when representing the patterns with equations. The lesson is useful for reinforcing for students that variables represent numbers and that various symbols, such as letters or shapes like boxes or triangles, can be used.

Students may suggest different equations to represent the same mathematical idea. For example, one student might describe the pattern as $(4 \times \square) + 2 = \triangle$, while another might describe it as $(2 \times \square) + [2 \times (\square + 1)] = \triangle$. Recognizing that there are many ways of representing one idea algebraically helps students learn about equivalent equations.

If you haven't had a good deal of experience with patterns like these, try investigating the problems in the "Extensions" section. Following are equations for each of them. Keep in mind that there is more than one way to represent these patterns with equations and you may see the pattern geometrically in a way that leads to a different algebraic representation. Also, don't provide these answers to your students. Let them work at their own level now. They will encounter these ideas many times as they continue their study of algebra.

A. $(4 \times \square) + 1 = \triangle$

B. $(2 \times \square) + 1 = \triangle$

C. $(4 \times \square) + 1 = \triangle$

D. $\square + 2 = \triangle$

E. $(2 \times \square) - 1 = \triangle$

F. $4 + (\square - 1) = \triangle$

G. $(\square \times \square) + 1 = \triangle$

VOCABULARY

axes, axis, column, coordinate graph, equation, equivalent equations, linear, ordered pair, pattern, plot, point, row, T-chart, variable

MATERIALS

- color tiles, about 50 per pair of students
- centimeter graph paper, several sheets per student (see Blackline Masters)
- 1 chart-size sheet of one-inch graph paper or 1 overhead transparency of centimeter graph paper
- Optional: *Piles of Tiles* activity sheet, 1 per pair of students (see Blackline Masters)

TIME

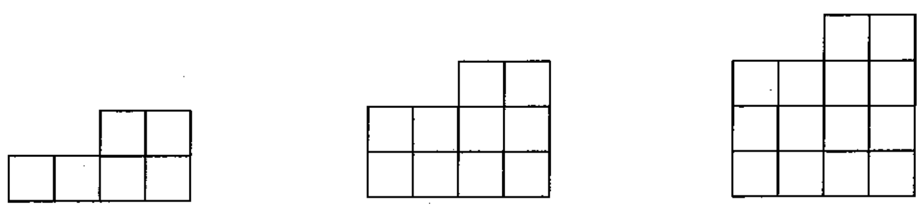
- at least three class periods

The Lesson

Day 1

Before class, I distributed a small plastic bag of approximately fifty color tiles to each pair of students. I said to the students, "Please leave the tiles in the bags for now. We'll use them soon."

I then built three piles of tiles on the overhead projector and numbered them 1, 2, and 3. I used regular tiles rather than overhead tiles to keep the students focused on the arrangements rather than on the colors used.



Pile 1 **Pile 2** **Pile 3**

I explained, "I built these three piles of tiles following a particular pattern. Please talk with your partner about how the fourth pile will look and then build your idea using tiles from your bag." The discussion was lively.

I circulated and listened to students explain their thinking, challenging those who finished more quickly to find a second way to think of the pattern. After a few minutes, I glanced around the room and noticed that most students had completed the fourth pile correctly. I asked for the students' attention and began a discussion.

A Class Discussion "Who would like to explain how to build the fourth pile?" I asked. About half the children had a hand raised. I called on Kurt.

Kurt explained, "You make four rows of four tiles and add two on top." He described what I had observed Tomo and Tim build. I wrote on the board:

Kurt Four rows of four and two on top

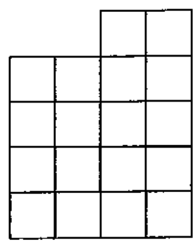
"Oh, wait," Kurt said after reading what I wrote. "The two tiles on top should be in the right corner." I revised what I had written:

Kurt Four rows of four and two on top in the right corner

"That's it," Kurt said.

"Who else thought of building Pile Four the same way as Kurt?" I asked. Several hands went up.

"Let me draw your idea on the board," I said. I sketched Kurt's idea, drawing a row of four tiles, then another row of four tiles above the first, then two more rows, and finally two tiles on top on the right. Kurt confirmed that was what he had done.



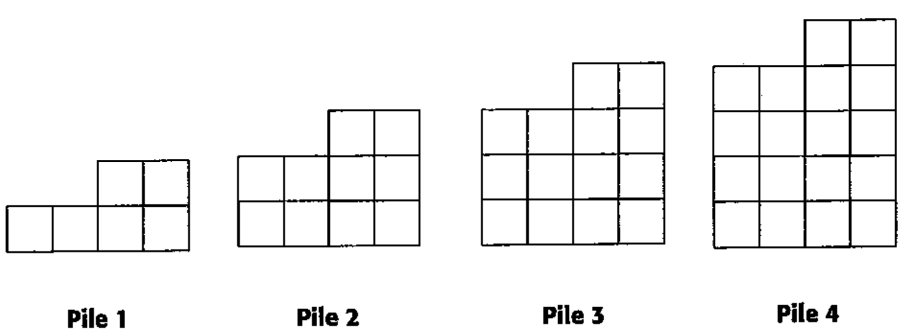
"Did anyone think about building the fourth pile a different way?" I asked.

Bradley explained, "You could do rows—no, I mean columns. You can do columns. You need two columns of four and two columns with five." Bradley's mistake of using the word *row* when he meant *column* is a common one. During the year the students participated in several lessons that involved the use of the words *row* and *column*. I was pleased Bradley caught his error. If he hadn't, I would have corrected him by restating his thought using the correct word.

I wrote Bradley's idea on the board under Kurt's:

Bradley Two columns of four and two columns of five

"Who would like to build the fourth pile on the overhead according to Bradley's idea?" I asked. "First I'll draw Piles One, Two, and Three on the board to make room on the overhead." I drew and labeled Piles 1, 2, and 3 to the left of Pile 4 so they were all in a row.



Many students were eager for the opportunity. I asked Bradley to call on someone and he called on Carmen. As Bradley read his directions, Carmen

built two columns with four tiles in each and then two columns with five in each. Bradley nodded his approval.

"It's still the same as Kurt's," Bradley commented.

"Yes, it's the same shape," I said and then asked, "Who has another way?"

Joanna explained, "You just add two rows of two on top."

I wasn't sure what Joanna meant, so I asked, "Can you come up to the overhead and show how you built it?" As Joanna was coming up, I cleared the tiles from the overhead projector and said, "The rest of you should build with your tiles as Joanna builds on the overhead. Joanna, please explain as you build so that the rest of the students can follow your directions with their tiles."

Joanna said, "First you build Pile Three." She stopped explaining to do this and I reminded the others also to build the third pile. "Then you add a row of two on top of these," Joanna said, placing a tile on the top of each of the two shorter columns. "And then you put two on top of these," she concluded, placing a tile on top of each of the two taller columns. The others did the same.

Joanna explained further, "See, these two columns are the same size, and so it's like adding a row of two. Then columns three and four both have one more tile than the first two, so I added one more to each of them and that was like another row of two because they were higher and the first two columns were lower."

"Joanna's way comes out the same," Alana commented. Several others nodded. There were no other comments. I wrote Joanna's idea on the board under the first two:

Joanna Add two rows of two to the top

"Does someone have another way?" I asked. Only a few hands went up. I called on Jerry.

"You could do one row of two and four rows of four," Jerry suggested. Jerry's idea was essentially the same as Kurt's, but he started building at the top, not the bottom. I recorded Jerry's idea under the others.

Jerry One row of two and four rows of four

"Is Jerry's like Kurt's, just turned around?" Rebecca asked hesitantly. Had Rebecca or one of the other students not raised this question, I would have addressed it with the class.

I said, "Yes, they're similar. They both looked at the rows and the two on top, but Jerry built from the top down and Kurt built from the bottom up." I left both ideas on the board to reinforce that there is more than one way to express the same idea. Also, I put a check next to each of their ideas and said, "I put these checks to show that they're similar."

Karena had another idea. She said, "I think my idea is like Joanna's, but I'm not sure. You could add a row of four to the bottom. Joanna added to the top, and I'm just thinking of adding it to the bottom instead. Either way, it's adding a row."

I wrote Karena's idea on the board under the others:

Karena Add a row of four on the bottom

"Talk with your partner about whether you think Karena and Joanna have the same idea." I said. The room came alive with conversation. In a

moment, hands began to go up as students made their decisions. When I called for the students' attention, almost all hands were in the air.

"Adam and I think they're the same," Geraldo said. "Like Karena said, you're adding a row of four either way."

"Penny and I think it's different because Joanna's way is two rows of two on top and Karena's way is one row of four on the bottom," Callie explained.

"But adding two rows of two like Joanna showed us is the same as adding one row of four," Rebecca argued.

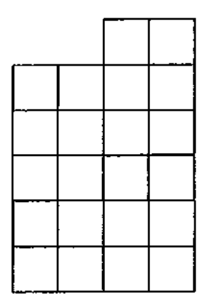
"I think that adding four on the top or four on the bottom is the same thing," Jon said. He had no explanation.

I said, "I see how you can look at it both ways." I left Karena's idea on the board. There were no other ideas for building the fourth pile.

Building Pile 5 I then said, "Some people saw the pattern of rows of four, as Kurt and Jerry did. Others looked at the columns, as Bradley did. And others saw the pattern as adding tiles on top or the bottom, as Joanna and Karena did. Now I'd like you to use these ways to build Pile Five. Take turns with your partner. First one of you chooses a way from the board and uses it to build Pile Five as your partner watches. Then your partner guesses which way you used. Then switch roles so the builder becomes the observer and the observer becomes the builder." The students were excited and got to work immediately. After a few minutes, I called the students to order.

"Raise your hand if you built Pile Five using Kurt and Jerry's idea about the rows," I said. About half the students raised a hand. I called on Tim and asked him to describe what he did. I followed his directions and built Pile 5 on the overhead projector. Tim and the other students agreed with what I did.

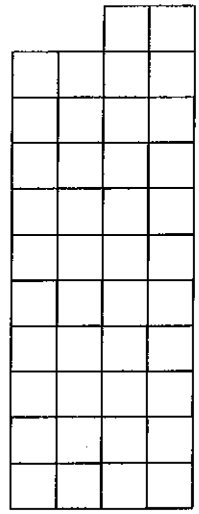
"Raise your hand if you used Bradley's column method to build Pile Five," I continued. About a fourth of the students raised a hand. I cleared the overhead and built Pile 5 as Rebecca gave directions. Again, the class agreed that I built Pile 5 correctly according to the column method.



The rest of the children indicated that they used Joanna's idea of adding one row to the previous structure. I again cleared the overhead and Penny explained how I should build Pile 5 using Joanna's idea.

Extending the Pattern "What about the tenth pile?" I said. "What would the tenth pile look like if I built it according to Kurt and Jerry's row method?" Hands went up immediately. I paused to give others time to think and then called on Tomo.

Tomo said, "You'd have ten rows of four and two more on top on the right." I built the tenth pile on the overhead projector.



"Thumbs up if you agree, thumbs down if you disagree, thumbs sideways if you're not sure," I said. Thumbs were up. "How do you know there should be ten rows?"

"That's easy," Tomo said. "In Pile One there was one row of four; in Pile Two, two rows; in Pile Three, three rows; and it goes on like that. So for Pile Ten there should be ten rows of four." Many students nodded in agreement. There were no other comments.

"How would I build the tenth pile according to Bradley's column method?" I asked.

Cara explained, "You'd have two rows of ten . . . no, I mean two columns of ten, and then two columns of eleven."

"Can I tell where the two columns of eleven come from?" Callie asked. I nodded. "The eleventh tile is from the extras at the top."

"I think the adding-on way is harder," Rebecca mused aloud.

"Why do you think that?" I asked her.

Rebecca explained, "Well, with the first two ways, you just know what to do and you can build the tenth pile with rows or columns. With the adding-on way, you have to build all the piles in between, Pile Five, Pile Six, like that, and add one row of four each time." Rebecca sighed and then added, "That seems like a lot of work to me!"

"I think I get what Rebecca's saying," Tim said. "You have to have the old pile before you can add a row of four to make the new pile. The first two ways are like a shortcut." Several students nodded and there were no more comments.

"What about the hundredth pile?" I asked. Eyes got big as the students thought about this for a moment. Then hands shot into the air as the students grasped how to describe the hundredth pile.

Penny explained, "You need to make one hundred rows of four and then add two on top. It's easy and it works!"

"Penny used the row method. What about the column method?" I asked.

Tim explained, "You'd need two columns of ten and two columns of eleven." I reminded Tim that I was asking about the hundredth structure. He looked confused.

"This is what the tenth pile looks like," I said, pointing to what I had built on the overhead projector. "The tenth pile has two columns of ten and

two columns of eleven. I want to know about the hundredth pile." Tim's eyes began to light up.

"Oh, I see now," Tim said. "It's two columns of a hundred and two columns of one hundred one!"

"Use your thumb to show if you agree or disagree with Tim," I said. All thumbs were up in agreement with Tim.

"What about the adding-a-row-of-four way?" I asked. Several students moaned. "Why are you moaning?" I asked.

"That would take forever because you'd have to build all the piles before it!" Callie said.

"Well, if you had the tenth pile, like we have right now, couldn't you just keep adding rows of four to it until there were one hundred?" Geraldo asked.

"How would you know about many rows to add?" I asked Geraldo.

"Well, you'd have to know that the pile number tells how many rows of four should be in the pile, so I guess in a way that would be like the row method." Several students nodded.

Making a T-Chart I drew a T-chart on the board and labeled the columns *# of Piles* and *# of Tiles.*

# of Piles	**# of Tiles**

I pointed to the sketches on the board of the first four piles and asked, "How many tiles do we need to build Pile One?"

"Six," the students chorused. On the T-chart, I wrote *1* in the left column and *6* in the right column. I didn't write these numbers at the top of the chart but left room so that later I could write numbers above them.

"How many tiles are needed to build Pile Two?" I continued.

"Ten," the class responded. I recorded *2* and *10*.

# of Piles	**# of Tiles**
1	6
2	10

"You can figure it out by multiplication," Rebecca shared. "Two times four is eight and the two on top make ten."

"So for the first pile, would it be one times four plus two?" Callie asked. I nodded. She smiled, pleased with her insight.

"How many tiles for Pile Three?" I asked.

"Fourteen," the class replied.

"Who can explain?" I asked. Almost all hands were up.

Chase explained, "There are three rows of four in Pile Three. I know because the pile number and the number of rows are the same. So three times four is twelve. Then you have to add the two on top, and that makes fourteen."

"Hey, I just noticed something," Adam said with surprise. "All the numbers in the tiles column are even. I bet they'll all be even."

"So you'd be suspicious if I wrote thirty-nine or forty-three on the right side of the chart?" I asked. He nodded.

"I think I know why what Adam says is true," Karena said. "You're always adding four. The first pile had six tiles and that's an even number, and four is an even number, so no matter what, you'll always get an even number because an even number added to an even number is an even number."

I wrote Adam's and Karena's ideas on the board near the T-chart:

Adam *All the numbers in the # of Tiles column are even.*
Karena *The # of Tiles numbers will always be even because four is always being added to another even number.*

I added *3* and *14* to the T-chart, then asked the students how many tiles would be needed for Pile 4. Carmen said, "Eighteen, because you need two columns of four and two columns of five. Two times four is eight and two times five is ten. Ten and eight equal eighteen."

"Carmen figured it out using my way of columns," Bradley commented.

The class agreed with Carmen and I recorded *4* and *18* on the T-chart. After I asked about the fifth pile and recorded *5* and *22* on the chart, I drew three dots under the *5* and wrote *10* and three dots under the 22 and wrote a question mark.

# of Piles	# of Tiles
1	6
2	10
3	14
4	18
5	22
.	.
.	.
.	.
10	?

I explained, "The dots mean I've intentionally skipped some numbers. In the Number of Piles column, I skipped six, seven, eight, and nine. In the

Number of Tiles column, I also made some dots, which means I skipped some numbers. Then I made a question mark. Why do you suppose I did this?"

"We're skipping some piles so we're also skipping some numbers of tiles, and the question mark means you want to know how many tiles for the tenth pile," Sadako said.

"That's just what I was thinking," I responded. "How many tiles are needed for the tenth pile, and how do you know? Talk with your partner. Try to think of more than one way to figure it out." The students talked eagerly and with confidence about their ideas. Most looked at the board and used the ideas I had recorded to help make their case. After a few moments, I called the students to order.

Tomo began the discussion. "Tim and I think forty-two is the number of tiles. All you have to do is use Kurt and Jerry's idea about rows, and then it's really easy to figure out."

Tim continued the explanation. "For Pile Ten there would be ten rows of four. Multiplying by ten is easy: four times ten is forty. Then there are the two on top, so you add two to forty and that's forty-two." Most students indicated their agreement by putting their thumbs up.

"Did anyone figure it out a different way?" I asked.

Alana explained, "Me and Karena knew there were twenty-two tiles in Pile Five, so for Pile Ten, we added twenty-two plus twenty-two, and that's forty-four."

"Hey," Karena noticed. "That's not right. It should be forty-two."

The girls looked at each other surprised, then started whispering. Alana exclaimed, "I know why! The twenty-two from Pile Five already has the two on top. We added the two at the top twice. That's why!"

"It's forty-two," Karena confirmed. There were no other comments. I erased the question mark and wrote *42*. Then I added some more dots at the bottom of both columns, wrote *100* in the # of Piles column, and wrote a question mark next to it in the # of Tiles column.

# of Piles	# of Tiles
1	6
2	10
3	14
4	18
5	22
.	.
.	.
.	.
10	42
.	.
.	.
.	.
100	?

"How many tiles are needed for the hundredth pile?" I asked. Hands flew into the air as students were eager to share. "When I call on you, tell us how many tiles you think are needed for the hundredth pile and how you thought about it. The rest of us will listen carefully and think about which method you're using to figure."

Tim shared first. He said, "You need two columns of one hundred and two more columns of one hundred plus two extras for the top. That makes four hundred two tiles." I wrote an equation on the board to represent Tim's thinking:

Tim $(2 \times 100) + (2 \times 100) + 2 = 402$

Sadako said, "That's Bradley's way. Another way of saying what Tim said is two columns of one hundred and two columns of one hundred one." I recorded:

Sadako $(2 \times 100) + (2 \times 101) = 402$

"I see another way!" Bradley blurted in excitement. He continued after apologizing, "This is really cool! Take Tim's way. If you put the two times one hundreds together, you get four times one hundred. Then there's the two on top. Four times one hundred is like the row way that Kurt and Jerry thought of. "

"Hey!" Dana said, "I did the row way. Four times one hundred plus two, and Bradley's right. They're the same!" I added Dana and Bradley's idea to the list:

Dana and Bradley $(4 \times 100) + 2 = 402$

"How many tiles would be needed for the thirty-fifth pile?" I asked.

Jon quickly replied, "Thirty-five. You'd have seven rows of five." Hands went up. I called on Alana.

"I disagree," Alana began. "Jon figured out how to make a rectangle with thirty-five. But in the pattern you have to have thirty-five rows of four and two on top."

"Oh yeah!" Jon said and hit his forehead with his hand. "Or you could have two columns with thirty-six and two columns with thirty-five."

"I agree," I said. "How many tiles would that be?" Jon shrugged. "Can I call on someone?" I nodded. Jon called on Tim.

"It's thirty-five times four," Tim said. "I know that thirty-five and thirty-five makes seventy. Seventy and seventy is one hundred forty, and then add two more for the top, and it's one hundred forty-two."

I wrote on the board:

Tim
$(4 \times 35) + 2$
$35 + 35 = 70$
$70 + 70 = 140$
$140 + 2 = 142$

I added *35* and *142* to the T-chart. I ended class and left the T-chart on the board for the next day.

# of Piles	# of Tiles
1	6
2	10
3	14
4	18
5	22
.	.
.	.
.	.
10	42
.	.
.	.
.	.
35	142
.	.
.	.
.	.
100	402

Day 2

To begin the lesson, I drew the students' attention to the T-chart from the day before and explained, "Work with your partner. You need to draw the T-chart and fill in the numbers for Piles One, Two, Three, all the way to Ten. Don't skip any of the numbers as I did yesterday. Record the number of tiles for each. Then write about the patterns you notice in the T-chart. The last thing for you to do is figure the number of tiles needed for Pile Twenty-Seven and explain how you figured it out."

"Do we go past Pile Ten?" Chase asked.

"You may if you wish, but you can stop at Pile Ten," I said.

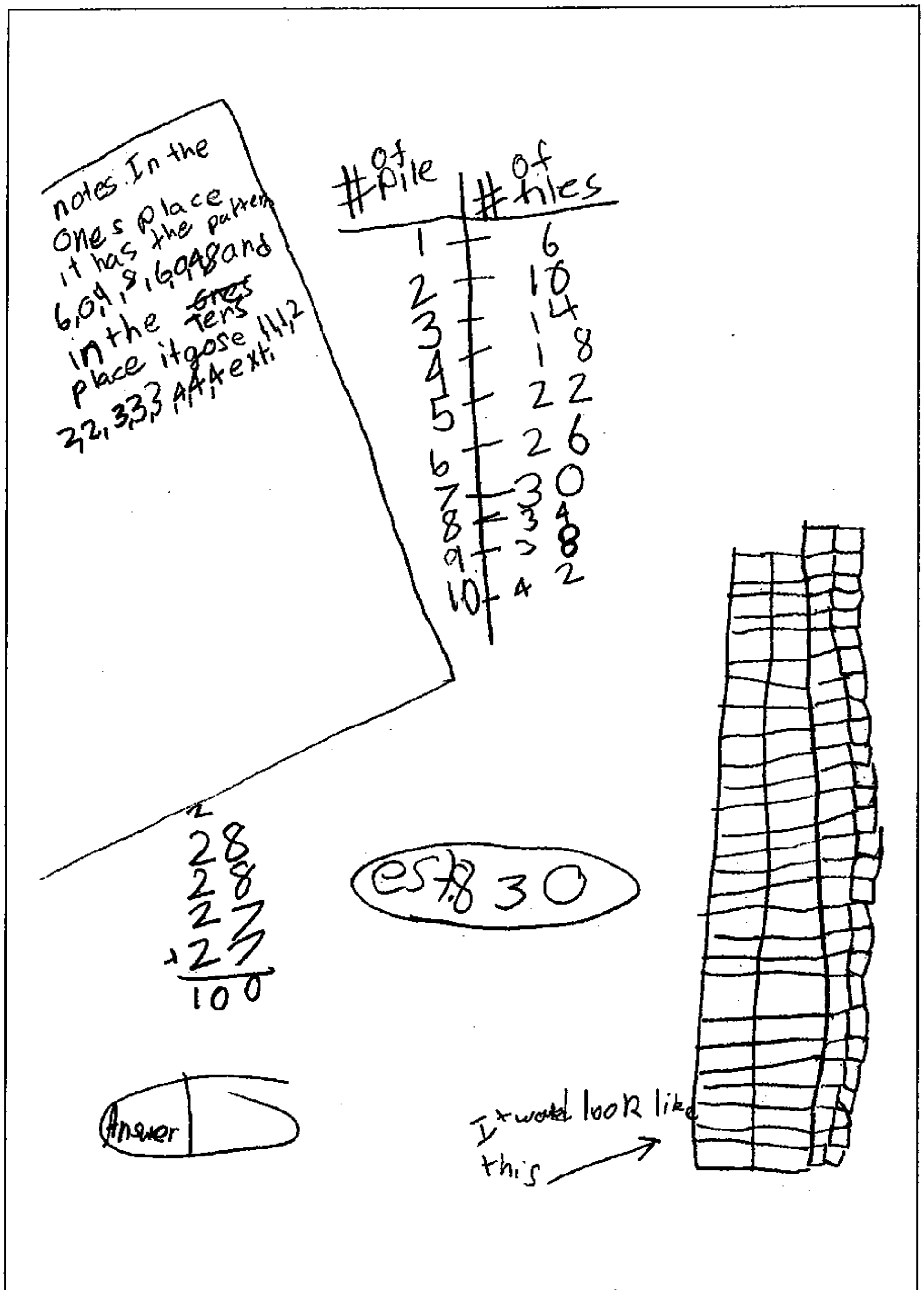
"Do we each write on our own paper?" Dana asked.

I replied. "Yes. Talk with your partner and share your ideas, but I'd like each of you to show your work on your own sheet of paper." There were no other questions and the students got to work. I quickly wrote the following directions on the board as a reminder to the students:

1. *Fill in the T-chart to the tenth pile.*
2. *Write about the patterns you see in the numbers on the T-chart.*
3. *Figure the number of tiles needed for Pile 27. Show how you know.*

As I circulated through the class, I reminded a few students to label the columns on their T-charts, answered a few questions, and stopped to talk with some students about their thinking. The students had little trouble with the task. As students finished, I checked their work and, when needed, asked them to make either corrections or clarifications. (See Figure 11–1.) I gave some students an additional challenge of figuring out the number of tiles for another pile number. I soon gave a one-minute warning to the students and then asked for their attention.

FIGURE 11–1 Lizzie illustrated the twenty-seventh pile correctly but made an addition error and didn't get the right answer.



A Class Discussion So that we had a reference for our discussion, I had the students help me complete the T-chart to the tenth pile.

# of Piles	# of Tiles
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34
9	38
10	42

Then I asked, "Who would like to share a pattern?" Almost all hands were up.

Callie shared, "I noticed all the numbers are even."

I paused for a moment, then pointed to the 3 in the # of Pile column. "Is three an odd number or an even number?" I asked. Although I knew that Callie was referring to the numbers in the # of Tiles column, I wanted her to be precise.

"It's odd," Callie replied. "I mean all the numbers on the other side are even."

"I see," I replied. I wrote on the board next to the T-chart:

Callie All the numbers in the # of Tiles column are even.

"Is this what you mean?" I asked. She nodded.

Tim shared next. "The number of tiles always increases by four." I added Tim's pattern to the list:

Tim The number of tiles always increases by four.

"What you wrote is right," Tim acknowledged.

Cassie shared next. She said, "In the Number of Tiles column, all the numbers in the ones place go six, zero, four, eight, two, six, and so on. They go in that order." I added Cassie's idea to the list and asked if what I wrote was correct. She nodded.

Cassie In the # of Tiles column, the numbers in the ones place go 6, 0, 4, 8, 2, 6, . . .

"I know another pattern," Dana said. "The left column, where it says Number of Piles, well, it goes up by one." I wrote Dana's idea on the list:

Dana The # of Piles column increases by one.

"Is this what you mean?" I asked to be sure that my paraphrasing accurately represented her idea. Dana nodded.

Karena reported the last idea. "If you look in the Number of Tiles column, and then look in the tens place of all the numbers, there's a pattern. It goes three ones in the tens place, then two twos, and then three threes, and I think it would be two fours in the tens place, forty-two and forty-six. Then if we kept going the pattern would go on like that." I recorded Karena's idea:

Karena In the Number of Tiles column, the tens place has 3 ones, then 2 twos, then 3 threes, and 2 fours, and so on.

I said, "I noticed as you worked that most of you wrote about how the number of tiles increases by four as we move down the T-chart." I used my finger to point this out on the chart. Stopping at 42 I said, "If we go back up the chart, what happens?"

"You're subtracting four," the students said. Starting with 42, we tested this idea by going up the chart and subtracting four each time. When we got to Pile One, with six tiles, I said, "The next pile up would be Pile Zero. If we subtract four from the six tiles for Pile One, how many tiles will that leave for Pile Zero?"

There was a flurry of conversation. "A zero pile is too weird."

"What's a zero pile?"

"You can't do that!"

"Yes, you can," Kurt said. "There are zero rows of four, and that's zero, and the two is the two left on the top." Again there was a flurry of conversation.

I settled the class and asked, "What do you think about Kurt's idea?"

Rebecca said, "I think he's right. The number of rows is always the same as the number of the pile, so the zero pile would have zero rows of four and just the two on top."

"I don't think so," Tomo said. "It doesn't make sense to have a zero pile. We have the first pile, the second, the third, like that, but there's no such thing as a 'zeroth' pile." The others giggled.

"Who's right?" Cara asked.

"In a way, they're both right. Kurt figured out a mathematical way to extend the chart that fits the pattern, so it's mathematically correct. But Tomo has a point, too. As he said, we don't think of a 'zeroth' pile." He was thinking in ordinal numbers.

"So is it OK to write zero and two on the chart?" Alana wanted to know.

"If you're thinking about the pattern mathematically, yes, as long as you know that you're not thinking about the situation of building with the tiles. What if we followed the chart up further and I wrote negative one above the zero? Then I have to subtract four from two to figure out the number to write next to negative one."

"Now that's really weird," Bradley said. "It's negative two, and you can't have negative two tiles."

"I bet if you could have negative two piles then you'd have negative six tiles," Penny conjectured. Many students nodded their agreement. I added this information to the T-chart and also drew a wavy line just below the zero and two:

# of Piles	# of Tiles
−2	−6
−1	−2
0	2
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34
9	38
10	42

I then said, "Remember, these numbers are mathematically correct, but there's a loss of reality above the wavy line. Keep in mind that some of the numbers we can write on the T-chart don't relate to the actual tiles."

Number of Tiles for Pile 27 To change the direction of the conversation, I said, "I asked you to find the number of tiles needed for Pile Twenty-Seven. How many are needed?"

Matthew said, "I think one hundred eight."

"I think that's wrong," Dana said. "Twenty-seven times four is one hundred eight, but there are the two tiles on top. You can't just ignore them. The answer should be one hundred ten." Others agreed. I wrote on the board:

$(27 \times 4) + 2 = 110$

"I did it differently," Tim said. "There's forty-two tiles in the tenth pile, so I added forty-two and forty-two to get the tiles in the twentieth pile. That's eighty-four. There's seven from twenty to twenty-seven, so I looked at the T-chart for Pile Seven and it says thirty tiles. I added eighty-four and thirty and got one hundred fourteen. What happened?" (See Figure 11–2 on the following page.)

"Oh, I know," Adam volunteered. "It's like what Karena and Alana did. You added the top two tiles twice and you should've only added them once."

Some students looked confused, so I tried to explain with smaller numbers so that the students could visualize the tiles. I said, "Suppose we think about smaller numbers, say Piles One and Two. Pile One needs six tiles." I drew Pile 1 on the board. "If we figure out the number of tiles for Pile Two by doubling the six tiles in Pile One, we get twelve." I drew Pile 2 on the board, then asked, "How many tiles do I really need for Pile Two?"

FIGURE 11–2 Tim got an incorrect number of tiles for Pile 27 by adding the number of tiles in Pile 10 to itself and then adding on the number of tiles in Pile 7.

# of pile	# of tiles
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34
9	38
10	42

$$\begin{array}{r} 42 \\ \times\ 2 \\ \hline 84 \\ +\ 30 \\ \hline 114 \end{array}$$

6,0,4,22.

4 keeps on adding no matter what

multiply 42 by two because in pile ten there are 42 tiles, you will get 84. Now add 30 because there are 30 tiles in the 7th pattern. you will get 114. That is how many tiles that are in pile 27.

"Ten," the class replied.

"Oh," Tim said. "If I just multiplied Pile One by three to get the number of tiles for Pile Three, that wouldn't work. It would be eighteen tiles and Pile Three needs fourteen."

Chase then returned to the twenty-seventh pile. He said, "I added twenty-seven and twenty-seven and twenty-eight and twenty-eight and it was one hundred ten."

I wrote Chase's idea on the board:

$$27 + 27 + 28 + 28 = 110$$

No one had anything else to share. I added 27 and 110 to the T-chart. Figures 11–3 through 11–5 show how three students worked on this activity.

FIGURE 11–3 Bradley correctly figured the number of tiles in twenty-seven rows of four.

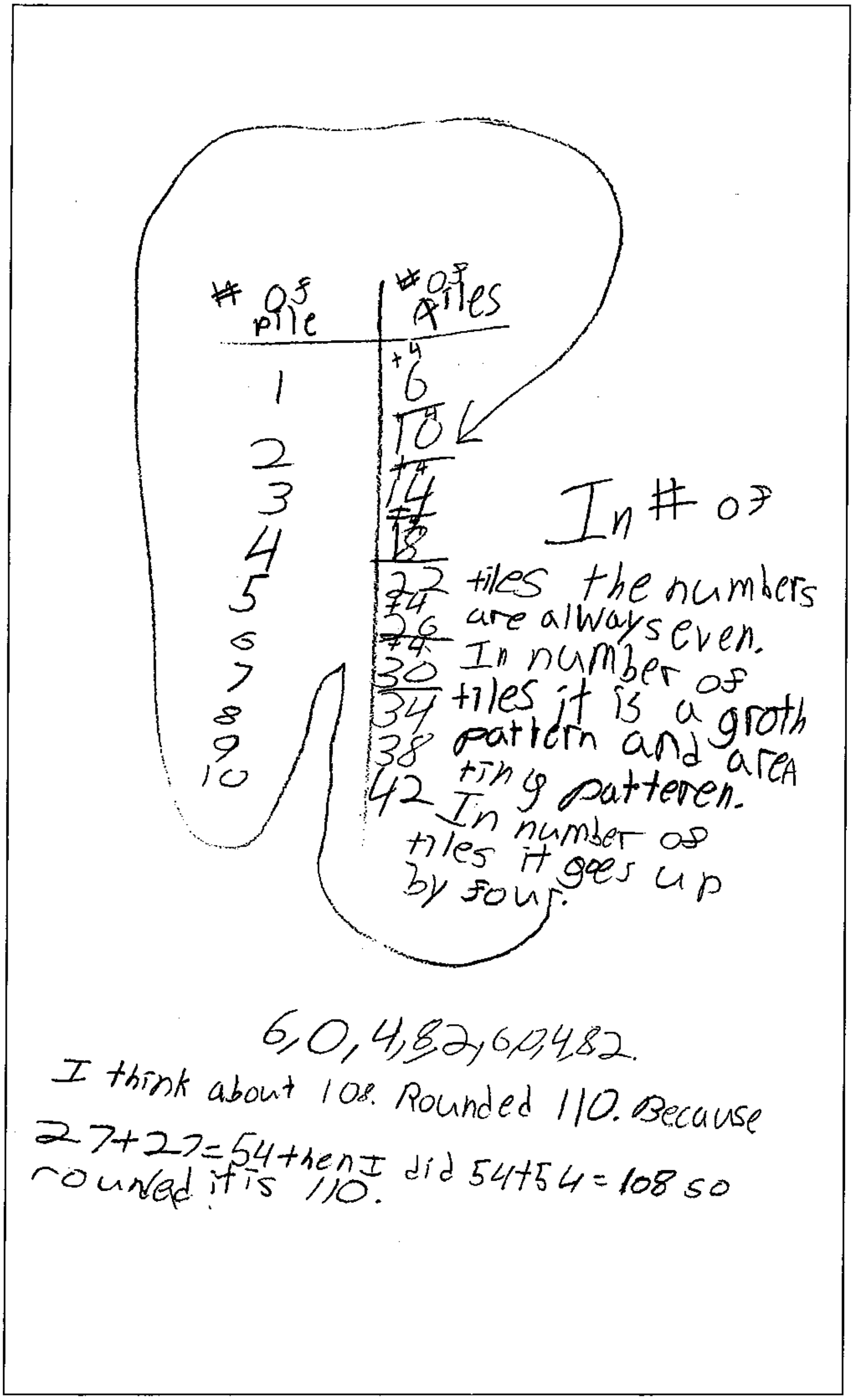


FIGURE 11-4 Tomo extended the T-chart to figure the number of tiles in the twenty-seventh pile. He also verified the answer with an equation.

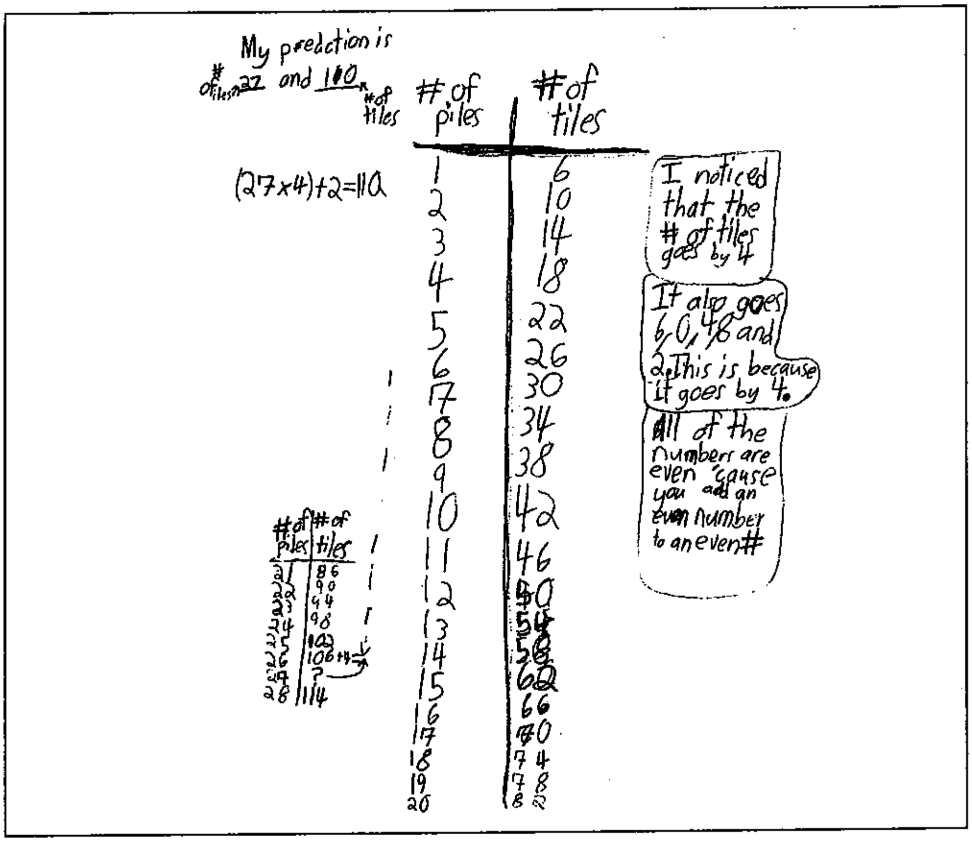
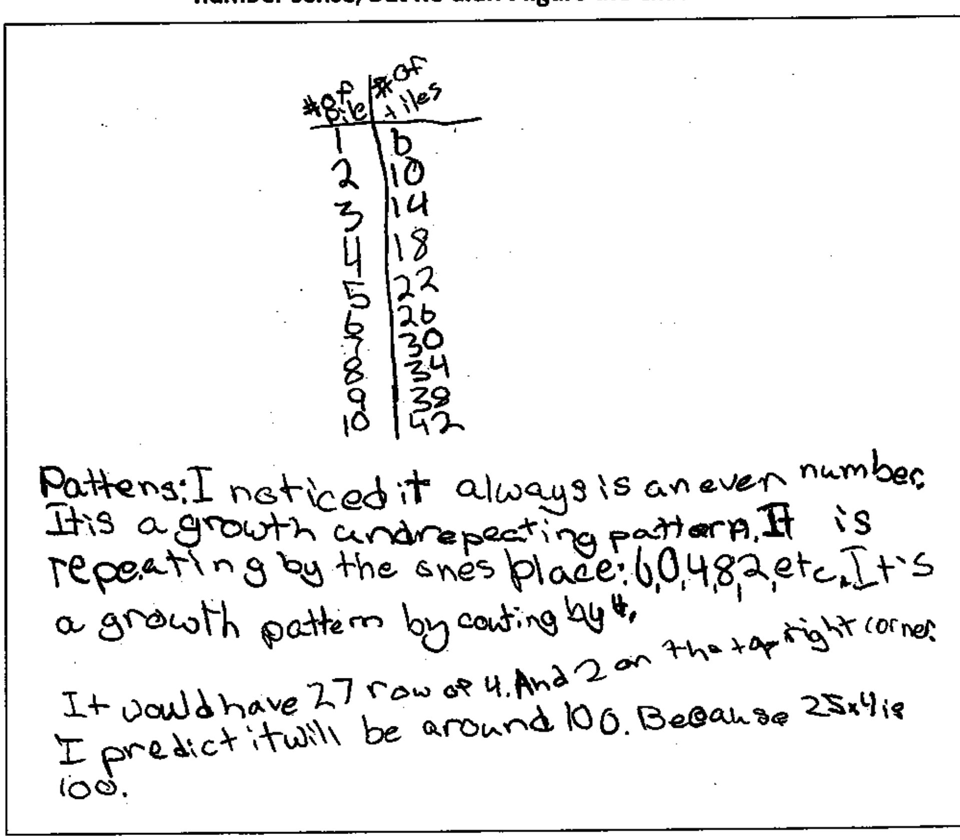


FIGURE 11-5 Chase's prediction for the number of tiles for Pile 27 shows good number sense, but he didn't figure the exact number of tiles.



Writing Equations Next I asked, "What if I wanted to know how to figure the number of tiles for any pile number? What would I do?" I paused to give the students a few moments to think. About half of the students had a hand up.

Jon said, "Well, for any pile number, multiply it by four, and add two to the product, and that equals the number of tiles."

"Why would you multiply by four and add two?" I pushed.

"The four is because there are four tiles in each row and the two is for the two tiles on the top," he explained. I wrote on the board:

Jon For any pile number, multiply it by 4, add 2 to the product, and that equals the number of tiles.

"I know another way," Bradley said. "Take the pile number, multiply it by two. Then take the pile number plus one and multiply that by two. Add the two numbers up. It's the column way and will give you the number of tiles."

I wrote on the board:

Bradley Take the pile number and multiply it by 2. Take the pile number, add 1, and multiply that by 2. Add the products and it equals the number of tiles needed for the pile.

There were no other comments. "It took me a long time to write Bradley's and Jon's ideas on the board," I said. "It would be much quicker if we wrote equations." I drew a box above the left column of the T-chart and a triangle above the right column.

□ **# of Piles**	△ **# of Tiles**
−2	−6
−1	−2
0	2
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34
9	38
10	42

I explained, "We'll let the box stand for the pile number and the triangle stand for the number of tiles. Let's look at Jon's idea." I pointed to Jon's idea. I read it aloud and wrote an equation, then put in parentheses for clarity:

$$(\square \times 4) + 2 = \triangle$$

"Sometimes mathematicians use letters for the variables instead of symbols," I said. "How could we write an equation with letters to describe Jon's idea?" Most hands were up.

Penny said, "You could use *p* for piles and *t* for tiles. Then it would be '*p* times four plus two is equal to *t*.' " I wrote *p* and *t* at the top of the T-chart above the box and triangle and then wrote on the board:

$$(p \times 4) + 2 = t$$

"These are two ways to write equations using variables for Jon's idea," I said. "Let's look at Bradley's idea and see if we can write an equation for it." I pointed to Bradley's idea and paused to give the students time to think. Hands went up quickly.

Dana said, "It's box times two, then box plus one times two, add those, then an equals sign and a triangle." I recorded:

$$\square \times 2 + \square + 1 \times 2 = \triangle$$

I said, "Dana's almost correct. Watch as I add some punctuation." I recorded:

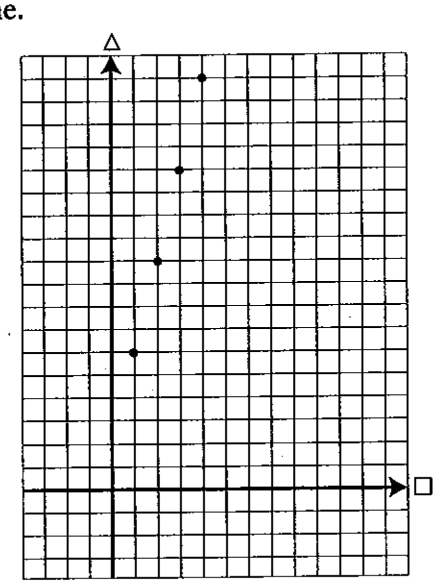
$$(\square \times 2) + [(\square + 1) \times 2] = \triangle$$

I explained, "We do what's in the parentheses first, and then the brackets."

Making a Graph I posted a large sheet of one-inch squared paper. An overhead transparency of centimeter graph paper can be used instead "We get to graph!" Joanna said.

"You're right," I replied. "First I'll draw the axes, and then let's look at the T-chart." I drew the axes.

"Let's start with the ordered pair (one, six). Who would like to mark the point for (one, six)?" The students were eager and their hands danced in the air. Cara came up, put the marker at the origin, counted over one and up six, and marked a point. Other students came up and plotted the next three ordered pairs. Some students commented about the pattern of the points going in a straight line.



Then I said, "What about the ordered pair (zero, two)? Will it fit the pattern?" Most thought it would. Sadako came up and marked it.

"Who's feeling brave and would like to mark the point for the ordered pair (negative one, negative two)?" Not as many hands were up; the students were less experienced with the other quadrants of the graph and how to graph negative numbers. I called on Javier. He came to the front and marked the point for (–1, 2) instead of (–1, –2). Several students disagreed.

Javier stepped back and shrugged. "It doesn't look right," he said.

I said, "Let's check. Starting at the origin, you counted to the left one for negative one. But then you went up. That would work for two, but not for negative two."

"Go down two," Kurt suggested.

"Oh, I see now," Javier said as he correctly marked the point.

Penny said, "Now that Javier moved his point, they all line up."

"That's correct," I said as I held a straight-edge along the points to show the line they made. "This is called a linear graph because the points make a line." I wrote *linear* on the board.

"I notice something else," Alana said. "If you go up the points like stairs, to go from one point to the next is four just like to go from one pile to the next is four."

"Oh yeah!" "Cool!" mumbled the students.

"I know what the four means!" Jon said with great excitement. "It's the number in each row of tiles! Wow! On the T-chart, the number of tiles goes up by four, and on the graph it goes up four!" There were no other comments and I ended the lesson.

Extensions

Students can work in pairs and explore patterns on their own. (See the *Piles of Tiles* activity sheet in the Blackline Masters.) For each, have the students follow these directions:

1. Build Piles 4 and 5 with tiles, then draw them on graph paper.
2. Make a T-chart and fill it in to the tenth pile.
3. Write about the patterns you notice in the tiles and on the T-chart.
4. Write an equation.
5. Make a graph.

Even though students work in pairs, you might want them to record individually so that they each get practice doing so.

You may want to assign the same pattern to everyone, or post the patterns and let students choose one that interests them (see the example at the top of page 220).

Lead class discussions for students to share their ideas and compare their T-charts and graphs. Figures 11–6 through 11–8 show how three students worked on this extension activity.

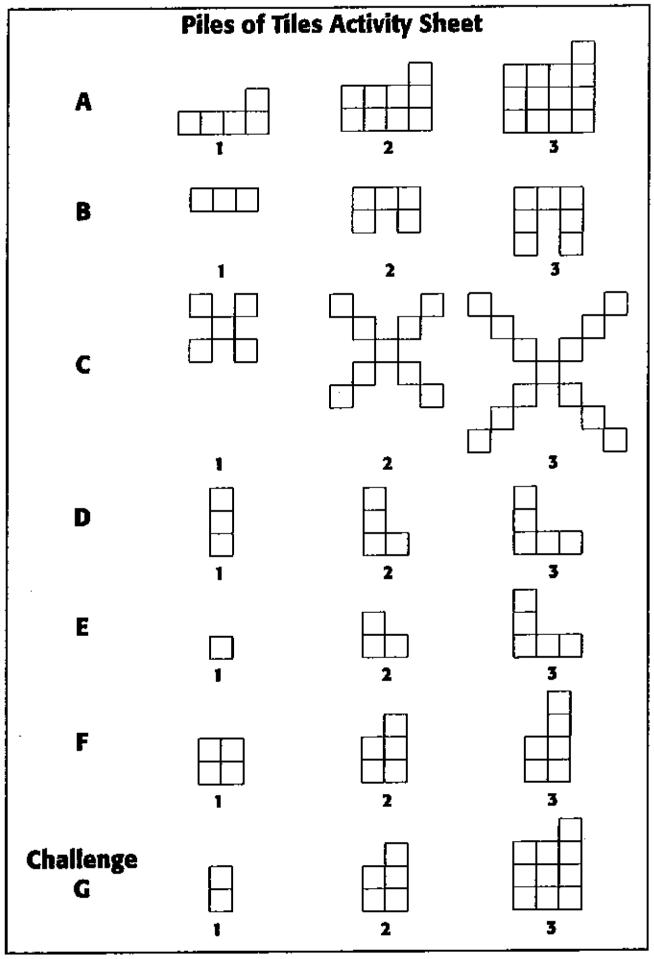


FIGURE 11–6 Cara successfully completed all parts of the investigation for Pattern B.

Pattern

The number of tiles is always odd. The number of tiles in the ones place end in: 3, 5, 7, 9, and 1. You add two to the number of tiles that you had before.

# of pile	# of tiles
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

Predict

I predict that #4 will have 9.

Equation:

2x ☺ +1 = ☺

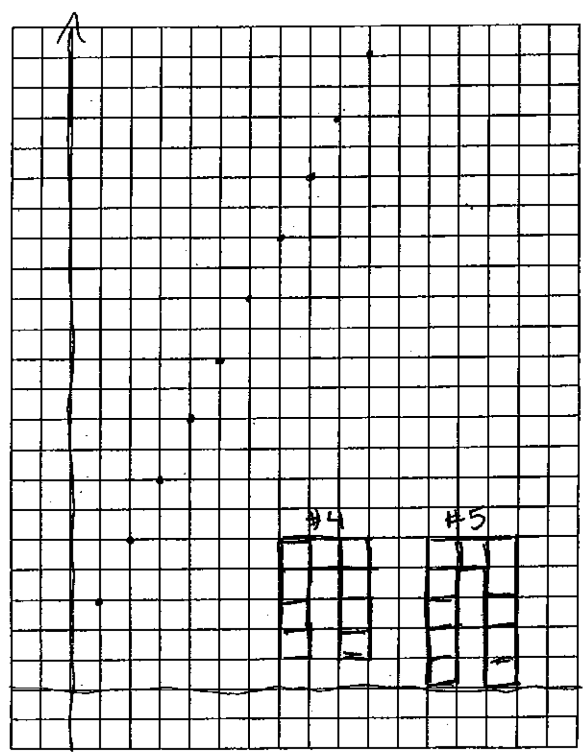


FIGURE 11–7 Jon investigated Pattern A, explaining how the pattern grew and graphing the point.

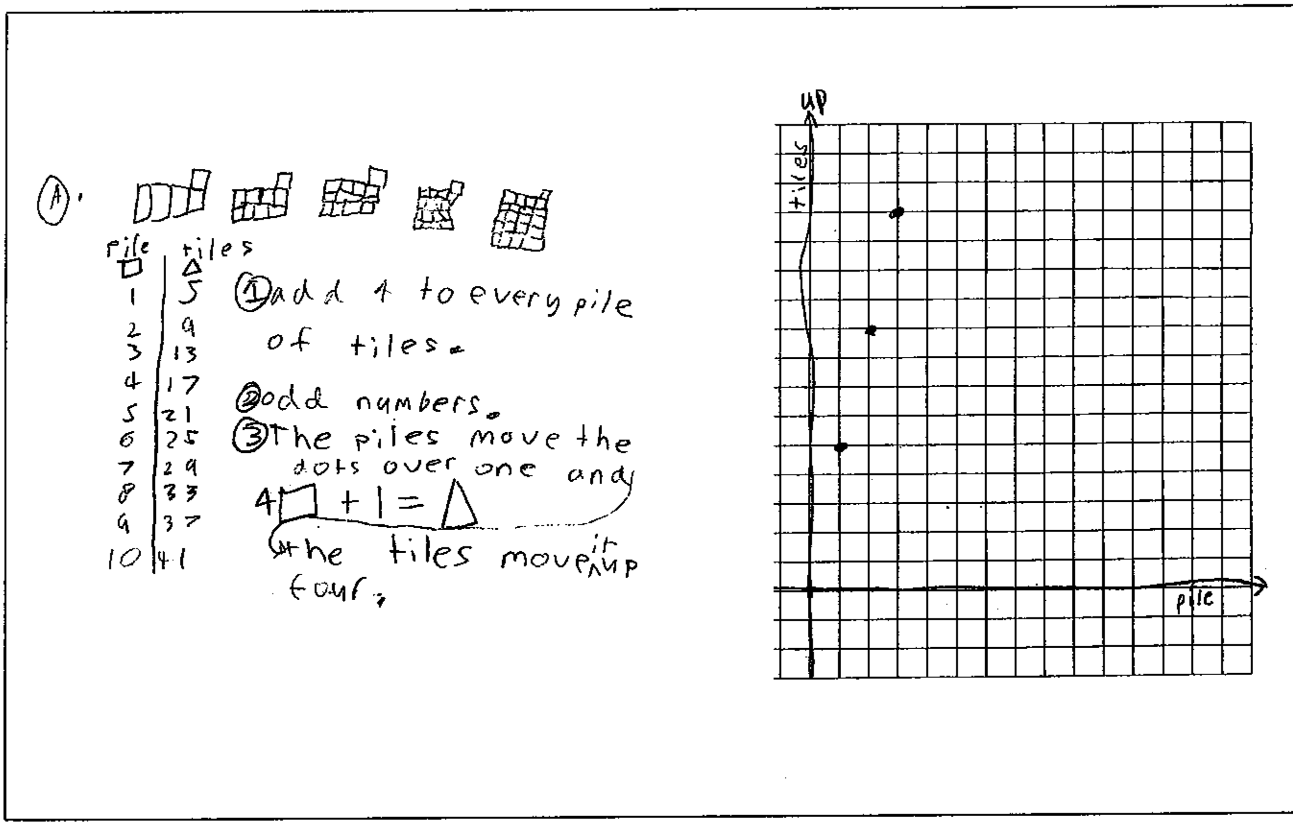
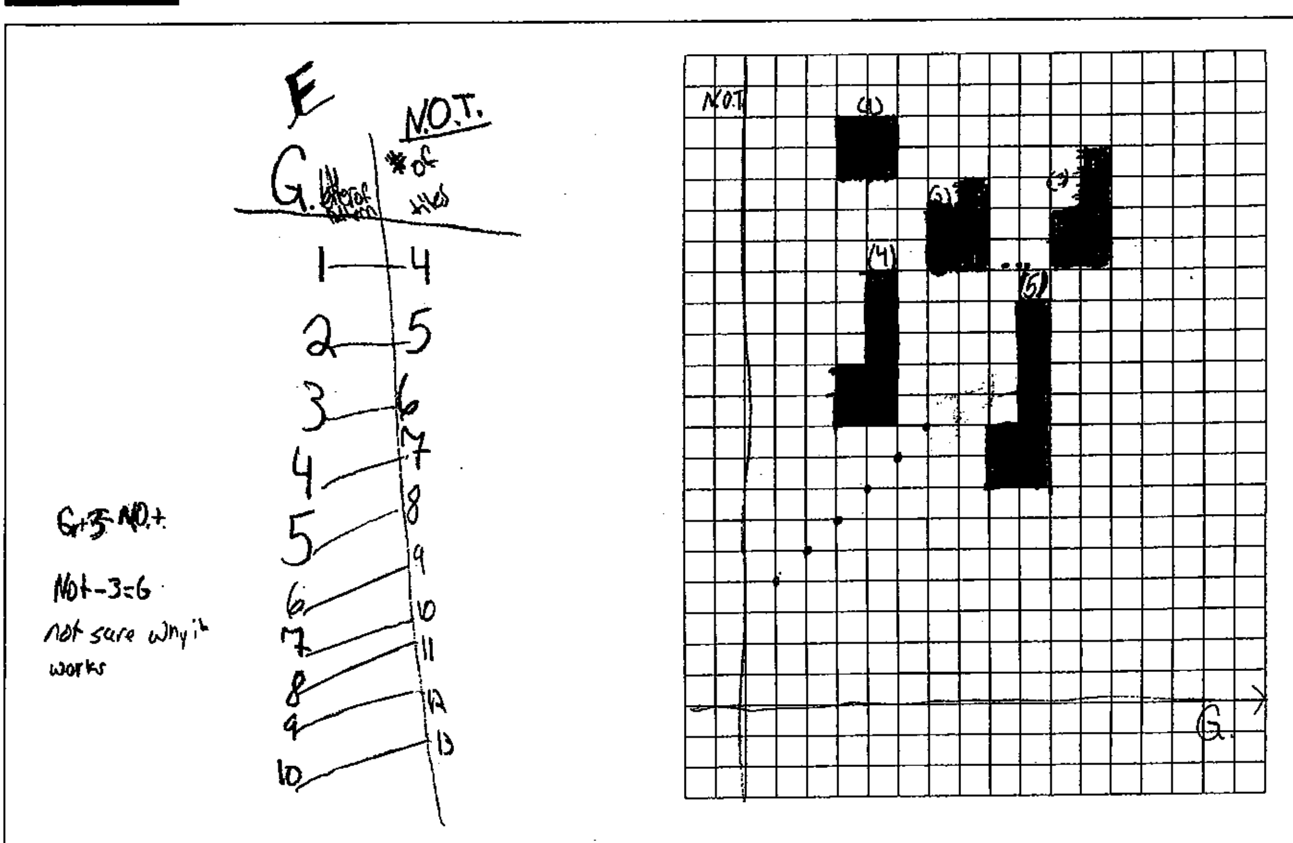


FIGURE 11–8 Dana discovered that the number of tiles in Pattern F is equal to the pile number plus 3.



CHAPTER 12

Table Patterns

Investigating Seating Arrangements

OVERVIEW

This lesson uses the context of seating people at tables in restaurants to investigate growth patterns. The students are first introduced to a restaurant where four people sit at each square table. After figuring out the number of people that can be seated at one, two, three, and so on tables up to ten, they figure out the number of people that can be seated when the tables are pushed together in a row for banquet-style seating. The students then repeat the investigation for tables that seat three, five, and six people each. They collect information about the patterns on T-charts, describe the rules, and write equations.

BACKGROUND

The McSquares, A Restaurant Tale of Tables and Algebra, a story by Marilyn Burns, provides the context for the investigations in this lesson. In the story, Mr. and Mrs. McSquare decide to open up a burger restaurant in their town. Everything in their restaurant is square—the sign, the tables, the plates, the bottoms of the cups, and even the burgers and buns. They start small, with three square tables seating four people each, but soon grow to ten tables. Also, they accommodate private parties, where they arrange the tables banquet style—one long row of tables pushed together. The McSquares are so successful that others are inspired to open their own restaurants. Mr. and Mrs. McTriangle specialize in grilled cheese sandwiches that they cut on the diagonal into triangles. The tables in their restaurant are triangles that seat three people each. Mr. and Mrs. McTrapezoid open a restaurant with trapezoidal tables that seat five people each, and Mr. and Mrs. McHexagon open a restaurant with hexagonal tables that seat six people each.

For each restaurant, students investigate the number of people that can be seated at from one to ten tables set individually, restaurant style. Then they investigate the number of people that can be seated at from one to ten tables when the tables are pushed together into a long row, banquet style.